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AN ANALYSIS OF FREE-CONVECTIVE HEAT TRANSFER FROM AN ISOTHERMAL VERTICAL PLATE TO SUPERCRITICAL FLUIDS

KANEYASU NISHIKAWA† and TAKEHIRO ITO‡
Department of Mechanical Engineering, Kyushu University, Fukuoka, 812 Japan

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Abstract—Theoretical studies have been made on the laminar free-convective heat transfer from an isothermal plate to fluids at supercritical pressures, taking into account of the temperature dependence of all the relevant physical properties. The heat-transfer characteristics of water at 230, 240 and 250 atm and carbon dioxide at 80 and 90 atm were clarified by integrating the similarity transformed differential equations numerically. Results were as follows:

(1) The heat-transfer characteristics are strongly dependent on the bulk fluid temperature and the wall temperature individually.
(2) For the larger temperature difference the conventional equation of laminar free-convective heat transfer fails to correlate the present analytical results.
(3) The analysis by Fritsch and Grosh for temperature-dependent density and specific heat compares favorably with the present one.

Property-dependence of the heat-transfer characteristics of the supercritical fluids may be said to have been thoroughly examined.

NOMENCLATURE

c_1, a quantity, equation (14);
c_2, a quantity, equation (15);
c_p, specific heat;
F, dimensionless velocity function, equation (8);
g, acceleration due to gravity;
H, dimensionless temperature, equation (9);
$\overline{Nu}$, average Nusselt number, $\bar{\alpha}x/\lambda$;
$\overline{Nu}^*$, $\overline{Nu}/x^{\frac{3}{4}}$;
Pr, Prandtl number;
q, heat flux on the heating surface;
Ra, Rayleigh number, $g(\rho_\infty - \rho)\,\rho c_p x^3/\lambda\mu$;
Ra^*, Ra/x^3;
T, temperature;
ΔT, temperature difference, $T_w - T_\infty$;
u, velocity component in x-direction;
v, velocity component in y-direction;
x, y, co-ordinates.

Greek symbols
α, heat-transfer coefficient;
η, similarity variable, equation (7);
λ, thermal conductivity;
μ, viscosity;
ρ, density;
ψ, stream function, equation (6).

Subscripts
c, pseudo-critical condition;
m, mean film temperature;
w, heating surface;
x, local value at x;
∞, the value far enough from the heating surface.

Superscripts
$'$, differentiation with respect to η;
$\bar{\ }$, averaged value by the operation of $(1/x)\int_0^x \mathrm{d}x\ \times$.

† Professor of Mechanical Engineering.
‡ Associate Professor of Mechanical Engineering.

1. INTRODUCTION

By recent development in industrial power system, the necessity for more accurate understanding of the heat-transfer characteristics of supercritical fluid has increased. Meanwhile a lot of investigations, both theoretical and experimental, have been reported on free-convective and forced-convective heat-transfer characteristics of these fluids. As is generally known, since physical properties of these fluids are strongly and singularly dependent on temperature and pressure, it is natural that the mechanism of heat transfer should become complicated. Especially there is the problem in the singularity of heat-transfer characteristics of supercritical fluids near the pseudo-critical point (which is, in the present paper, arbitrarily defined as the point where the specific heat at constant pressure becomes maximum on an isobar). Though the discrepancy between the calculated and observed values can not be satisfactorily explained at present, one may attribute it to the lack of accurate data of physical properties, or to an incomplete analysis, in which the temperature-dependence of physical properties is not adequately taken into account, or to the occurrence of phenomenon similar to boiling.

Fritsch and Grosh [1] analysed laminar free-convective heat transfer from an isothermal vertical plate for a fluid with temperature-dependent density and specific heat. On the other hand, Hasegawa and Yoshioka [2] calculated the first perturbation solution to the non-perturbated solution for the fluid having physical properties at pseudo-critical point, expressing all the relevant physical properties as some power functions of enthalpy. These two analyses seem to coincide with each other and agree favorably with experimental results only in case of small temperature differences. For large temperature differences, however, the heat transfer coefficient estimated from these analyses is much smaller than those measured. At present it is difficult to determine whether the above-mentioned coincidence or discrepancy resulted from the difference of assumptions used to obtain solutions in these two analyses, or from the assumption of similar laminar boundary layer on which they were based.

In the authors' laboratory, researches in this field have been kept up since 1960, publicizing our views [3, 4] supporting seemingly the hypothesis of boiling-like phenomenon. To judge the validity of the hypothesis it is however necessary to obtain a theoretical solution in which the temperature-variation of physical properties is fully taken into account. This was the chief aim of this study, and the case of free-convection heat transfer from an isothermal vertical plate was analysed because it was considered easier to check the result of analysis by experiment than in the case of forced-convection system.

2. ANALYSIS

As shown in Fig. 1, the case studied here is a vertical isothermal plate from which heat is removed by steady laminar free-convection. The plate is kept at a uniform temperature T_w, and the bulk fluid T_∞. A laminar boundary layer is assumed to develop along the plate. The conservation laws for mass-, momentum- and energy-transfer will be as follows, if all the relevant physical properties vary with temperature.

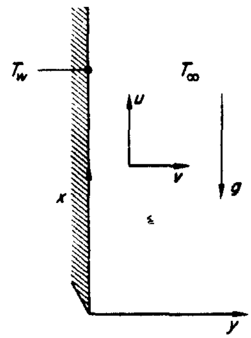

Fig. 1. Physical model and co-ordinates.

$$\frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = g(\rho_\infty - \rho) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{2}$$

$$\rho c_p \left(u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y}\right) = \frac{\partial}{\partial y}\left(\lambda \frac{\partial T}{\partial y}\right). \tag{3}$$

Boundary conditions are:

$$y = 0: \qquad u = v = 0, \quad T = T_w \tag{4}$$

$$y \to \infty: \qquad u \to 0, \quad T \to T_\infty. \tag{5}$$

It is extraordinarily difficult, even for temperature-independent properties, to obtain the analytical solution for the set of non-linear differential equations of (1), (2) and (3) under the conditions of (4) and (5). The similarity relations will be assumed as in the case of Fritsch and Grosh [1] and Hasegawa and Yoshioka [2].

It can be shown that a stream function which satisfies the following relations exists also for variable density steady two-dimensional flow field:

$$\rho u = \rho_\infty \frac{\partial \psi}{\partial y}, \qquad \rho v = -\rho_\infty \frac{\partial \psi}{\partial x}. \tag{6}$$

u and v in equations (2) and (3) are replaced by ψ. Then the assumption of the existence of the similarity relations is applied to the flow- and temperature-fields. A similarity variable η is introduced as an extension of the case of fixed physical properties:

$$\eta = c_1 x^{-\frac{1}{4}} \int_0^y \frac{\rho}{\rho_\infty}\,\mathrm{d}y \tag{7}$$

where c_1 is some arbitrary constant. The above-mentioned similarity relations are defined for stream function F and the dimensionless temperature H as follows again as an extension of the case of fixed physical properties:

$$\psi = c_2 x^{\frac{3}{4}} F(\eta) \tag{8}$$

$$(T - T_\infty)/(T_w - T_\infty) = H(\eta) \tag{9}$$

where c_2 is some arbitrary constant. Physical properties are considered as functions only of temperature, therefore only of η. Substituting these transformation-relations into the fundamental equations (2) and (3) and the boundary conditions (4) and (5), the following equations are obtained:

$$[(\rho\mu/\rho_\infty\mu_\infty) F'']' - 2(F')^2 + 3FF'' + \rho_\infty/\rho - 1 = 0 \tag{10}$$

$$[(\rho\lambda/\rho_\infty\lambda_\infty) H']' + 3Pr_\infty(c_p/c_{p\infty}) FH' = 0 \tag{11}$$

$$\eta = 0: \qquad F = F' = 0, \quad H = 1 \tag{12}$$

$$\eta \to \infty: \qquad F' \to 0, \quad H \to 0 \tag{13}$$

where c_1 and c_2 are chosen optionally as follows in order to let them be identical with those solved by Ostrach [5] in the case when properties are independent on temperature:

$$c_1 = (g\rho_\infty^2/4\mu_\infty^2)^{\frac{1}{4}} \tag{14}$$

$$c_2 \doteq 4(g\mu_\infty^2/4\rho_\infty^2)^{\frac{1}{4}}. \tag{15}$$

After all the mathematical expression of the present problem is to solve the set of ordinary differential equations (10) and (11) with the boundary conditions of (12) and (13). These equations were solved by an electronic digital computer, applying successive approximation of difference equations. The local heat flux and the local heat-transfer coefficient at height x from the leading edge are calculated using the so determined dimensionless temperature H as follows:

$$q_x = -\lambda_w \left.\frac{\partial T}{\partial y}\right|_{y=0} = \lambda_w(T_w - T_\infty)(g\rho_w^4/4\rho_\infty^2\mu_\infty^2)^{\frac{1}{4}}\{-H'(0)\}\,x^{-\frac{1}{4}}$$

$$\alpha_x = q_x/(T_w - T_\infty) \tag{16}$$

$$= \lambda_w(g\rho_w^4/4\rho_\infty^2\mu_\infty^2)^{\frac{1}{4}}\{-H'(0)\}\,x^{-\frac{1}{4}}. \tag{17}$$

The average heat flux and the average heat-transfer coefficient are simply obtained by the relations:

$$\bar{q} = (1/x)\int_0^x q_x\,\mathrm{d}x = (\tfrac{4}{3})\,q_x \tag{18}$$

$$\bar{\alpha} = \bar{q}/(T_w - T_\infty) = (\tfrac{4}{3})\,\alpha_x \tag{19}$$

$q_x x^{\frac{1}{4}}$, $\bar{q}x^{\frac{1}{4}}$, $\alpha_x x^{\frac{1}{4}}$ and $\bar{\alpha}x^{\frac{1}{4}}$ do not include x because of the assumed similarity. All the illustrations and discussions of the results of analysis will be made in terms of these quantities.

3. RESULTS AND CONSIDERATION

Numerical calculation was performed on water at 230, 240 and 250 atm and carbon dioxide at 80 and 90 atm systematically. Besides, several cases were analysed for a comparison with the former solutions and experiments. In the systematic calculation the bulk fluid temperature was set equal to the following 11 values:

$$T_\infty - T_c = 0, \pm 1, \pm 2, \pm 3, \pm 4, \pm 5 \text{ deg.}$$

where T_c is the pseudo-critical temperature corresponding to each pressure. For each bulk fluid temperature the wall excess temperature was specified as following 7 ways:

$$\Delta T \equiv T_w - T_\infty = 0{\cdot}25, 0.5, 1, 2, 4, 8, 16 \text{ deg.}$$

Results of calculation are given in Table 1 for water, in Table 2 for carbon dioxide. In Table 1 there are five blanks lacking entries, because the process of successive approximation failed to give a converged solution.

3.1. *Heat flux and heat-transfer coefficient*

To illustrate the general feature of the results

Table 1. Results of calculation for water $\bar{q}x^{\frac{1}{4}}$ in kcal/h $m^{\frac{7}{4}}$

P (atm)	$T_\infty - T_c$ (deg)	T_∞ (°C)	$\Delta T = T_w - T_\infty$ deg: 0·25	0·5	1	2	4	8	16
	5	380·82	40·87	96·47	224·7	514·3	1142	2438	4883
	4	379·82	46·82	103·7	235·1	562·5	1258	2657	5256
	3	378·82	54·56	128·1	295·8	658·9	1436	2970	5751
	2	377·82	79·42	178·8	394·0	847·3	1756	3477	6545
	1	376·82	97·00	202·1	459·2	1026	2120	4108	7508
230	0	375·82	284·9	587·5	1149	2125	3814	6569	10940
(T_c = 375·82°C)	−1	374·82	161·8	394·6	1049	2744	5267	9028	14770
	−2	373·82	112·3	272·5	672·2	1754	4793	9090	—
	−3	372·82	97·74	233·4	564·1	1402	4051	—	—
	−4	371·82	87·61	209·2	501·6	1227	2989	8377	—
	−5	370·82	81·59	194·6	467·5	1131	2825	7849	—
	5	384·44	47·10	111·0	257·9	588·1	1299	2741	5448
	4	383·44	54·83	127·9	295·4	667·3	1457	3026	5915
	3	382·44	65·96	155·0	357·0	795·0	1693	3430	6558
	2	381·44	82·15	190·8	433·5	959·1	2016	3980	7408
	1	380·44	118·9	272·3	604·5	1283	2587	4904	8778
240	0	379·44	190·6	436·6	959·5	1968	3744	6800	11320
(T_c = 379·44°C)	−1	378·44	167·8	406·8	996·6	2317	4642	8281	13700
	−2	377·44	123·3	298·1	738·0	1893	4536	8663	14640
	−3	376·44	104·9	250·3	606·8	1513	3998	8587	14960
	−4	375·44	92·95	223·4	538·7	1318	3382	8306	15080
	−5	374·44	85·87	204·9	492·7	1198	3002	7847	15120
	5	387·98	53·11	124·6	288·9	657·4	1448	3041	6009
	4	386·98	60·72	142·4	329·8	745·8	1628	3365	6536
	3	385·98	72·88	170·1	390·8	874·9	1874	3799	7238
	2	384·98	90·46	211·0	481·9	1063	2230	4407	8178
	1	383·98	118·1	274·3	621·8	1352	2769	5296	9520
250	0	382·98	145·0	338·7	775·0	1697	3447	6440	11240
(T_c = 382·98°C)	−1	381·98	145·6	347·0	823·5	1893	3987	7480	12870
	−2	380·98	127·2	304·6	734·1	1778	4075	8019	13890
	−3	379·98	107·9	258·6	627·0	1549	3811	8138	14440
	−4	378·98	97·11	231·7	556·5	1361	3427	7959	14680
	−5	377·98	89·71	214·8	515·1	1246	3099	7646	15770

Table 2. Results of calculation for carbon dioxide $\bar{q}x^{\frac{1}{4}}$ in kcal/h m$^{\frac{7}{4}}$

P (atm)	$T_\infty - T_c$ (deg)	T_∞ (°C)	$\Delta T = T_w - T_\infty$ deg						
			0·25	0·5	1	2	4	8	16
	5	38·72	12·29	28·96	67·54	154·7	345·1	737·8	1495
	4	37·72	13·93	32·75	75·94	172·9	381·6	807·4	1612
	3	36·72	16·09	37·75	87·33	197·5	431·2	896·9	1761
	2	35·72	19·37	45·20	103·6	231·7	497·6	1015	1955
	1	34·72	28·65	65·87	145·7	310·3	633·2	1236	2289
80	0	33·72	49·93	110·4	233·1	470·1	896·8	1641	2891
(T_c = 33·72°C)	−1	32·72	40·07	97·85	253·6	594·7	1246	2077	5368
	−2	31·72	34·63	82·88	200·4	490·8	1154	2172	3771
	−3	30·72	27·75	66·71	162·8	410·7	1042	2167	3848
	−4	29·72	24·53	58·73	141·9	350·3	879·9	2111	3880
	−5	28·72	22·72	54·27	130·1	316·0	801·7	2012	3873
	5	44·52	13·98	33·03	77·02	176·1	392·8	844·8	1731
	4	43·52	15·55	36·65	85·38	195·1	431·5	915·6	1849
	3	42·52	17·85	42·01	97·73	221·9	485·5	1011	2004
	2	41·52	20·79	48·84	113·2	255·7	553·1	1131	2195
	1	40·52	26·75	62·69	143·7	316·7	666·1	1321	2485
90	0	39·52	34·75	81·03	184·4	402·4	827·8	1593	2895
(T_c = 39·52°C)	−1	38·52	36·35	85·87	201·0	456·6	959·9	1842	3289
	−2	37·52	35·38	83·93	199·3	465·0	1025	2015	3594
	−3	36·52	31·47	74·86	179·6	434·7	1009	2095	3787
	−4	35·52	28·77	68·53	164·2	397·5	961·5	2109	3903
	−5	34·52	26·21	62·42	149·6	363·8	897·0	2071	3970

heat-transfer characteristics of water at 240 atm are shown from Figs. 2–5, that of carbon dioxide at 80 atm from Figs. 6–9. It is clearly seen from these figures that both heat flux and heat-transfer coefficient are strongly dependent on the bulk fluid temperature and the wall temperature individually. This is a peculiarity remarkable for the heat transfer to or from fluid of temperature-dependent physical properties, which is essentially different from that of fixed physical properties where the role of temperature difference is predominant. For a fixed bulk fluid temperature and increasing temperature difference, plot of heat flux vs. temperature difference (Figs. 2 and 6) and that of heat-transfer coefficient vs. temperature difference (Figs. 3 and 7) gradually increase their slopes as the wall temperature approaches the pseudo-critical temperature.

For a fixed temperature difference and varying bulk fluid temperature, plot of heat flux verses bulk fluid temperature (Figs. 4 and 8), and that of heat-transfer coefficient versus bulk fluid temperature (Figs. 5 and 9) show maxima when the wall temperature and/or bulk fluid temperature approach the pseudo-critical temperature. These are thought to explain the singularity of heat transfer in some region near the psuedo-critical temperature. In these figures, the larger the temperature difference is, the higher the bulk fluid temperature which maximizes heat flux or heat-transfer coefficient is. It is however seen that the wall temperatures for such a situation (bulk fluid temperature plus temperature difference) similarly come close to the pseudo-critical temperature. Furthermore, the larger the temperature difference is, the flatter the curves become and heat transfer is not influenced by temperature difference so strongly as in the case of small temperature differences. This can be understood as follows. With larger temperature differences, heat transfer is considered to be determined by fluid of wider range of physical properties corresponding to the temperature

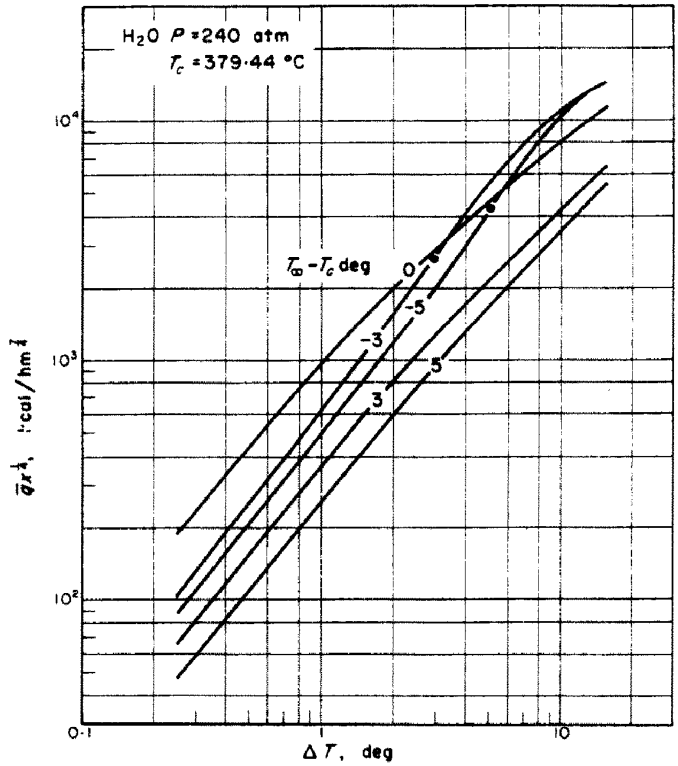

FIG. 2. Relation between $\bar{q}$ and ΔT (water, 240 atm). Wall temperature is equal to the pseudo-critical value at the points marked by "●". The same is said from Figs. 3–9.

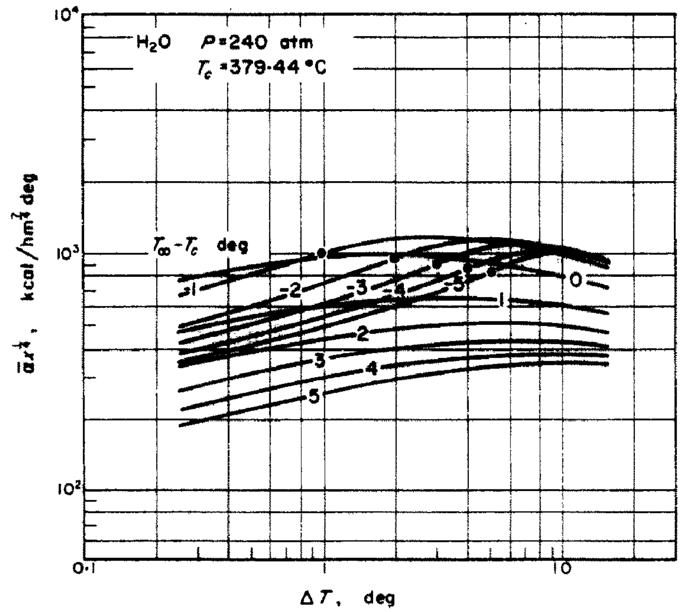

FIG. 3. Relation between $\bar{\alpha}$ and ΔT (water, 240 atm).

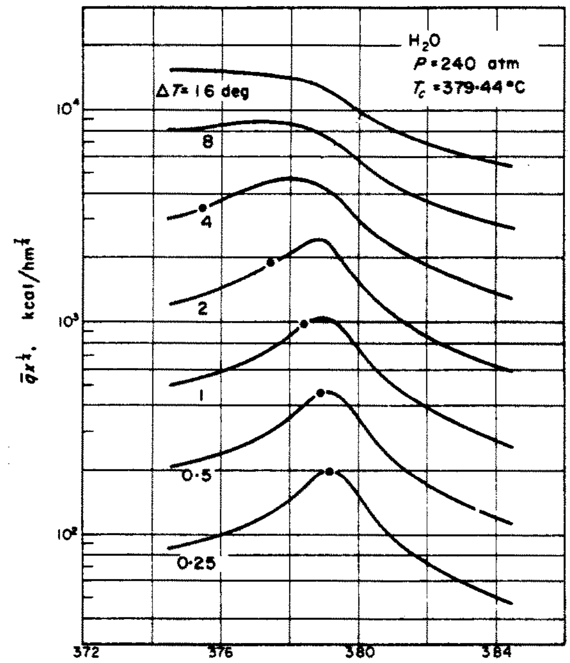

FIG. 4. Relation between $\bar{q}$ and T_∞ (water, 240 atm).

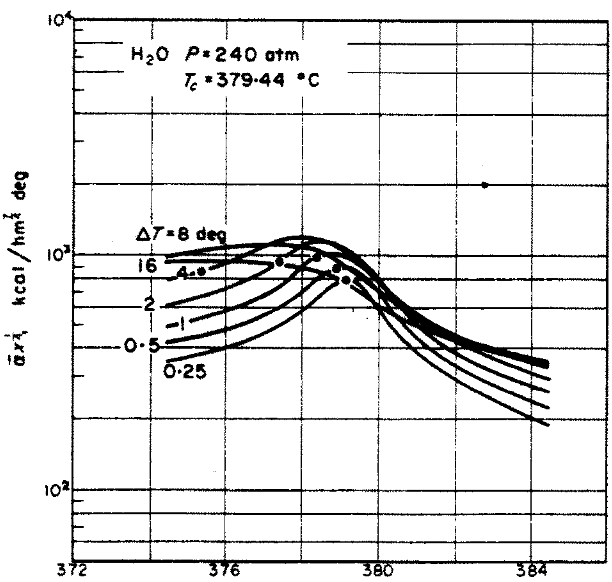

FIG. 5. Relation between $\bar{\alpha}$ and T_∞ (water, 240 atm).

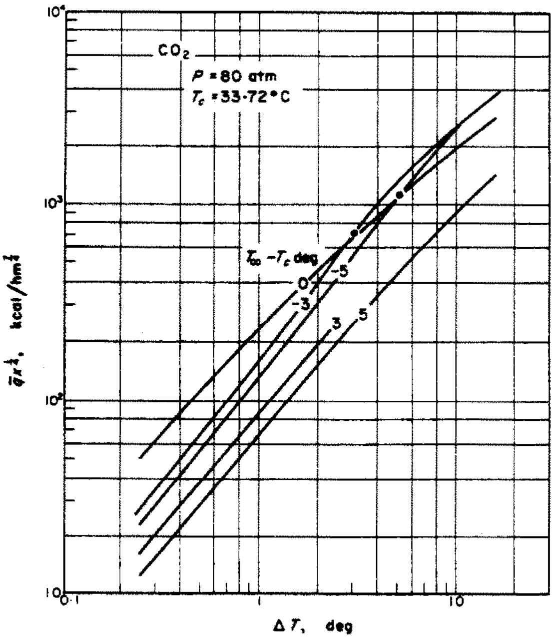

FIG. 6. Relation between $\bar{q}$ and ΔT (carbon dioxide, 80 atm).

range, and the singularities of variation of physical properties near the pseudo-critical temperature is not so directly reflected on heat transfer.

3.2. *Nondimensional representation*

As illustrated in the previous section it is rather hopeless to derive a correlation of heat transfer by dimensional analysis for such a situation as the present one where heat transfer to fluids of strong temperature-dependent properties is considered. It is however practically important to examine how the conventional correlation of laminar free-convective heat transfer taking the Rayleigh number *Ra* as independent variable and the average Nusselt number $\overline{Nu}$ as dependent variable fails to correlate the present results. Making choice of the mean film temperature $(T_w + T_\infty)/2$ as the reference temperature of physical properties, the calculated values were transformed as follows. The average heat-transfer coefficient $\bar{\alpha}$ multiplied by the fourth root of height of the

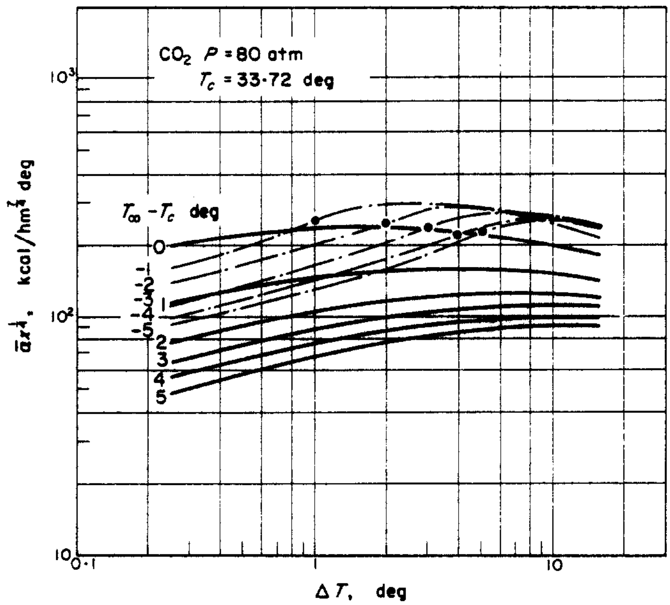

FIG. 7. Relation between $\bar{\alpha}$ and ΔT (carbon dioxide, 80 atm).

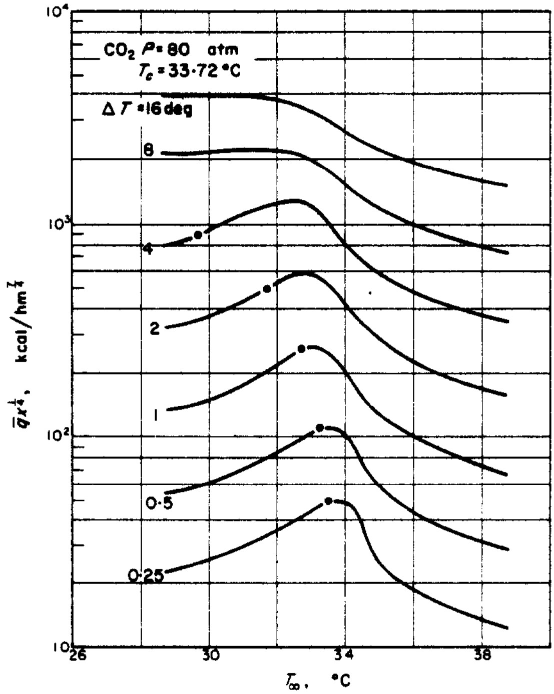

FIG. 8. Relation between $\bar{q}$ and T_∞ (carbon dioxide, 80 atm).

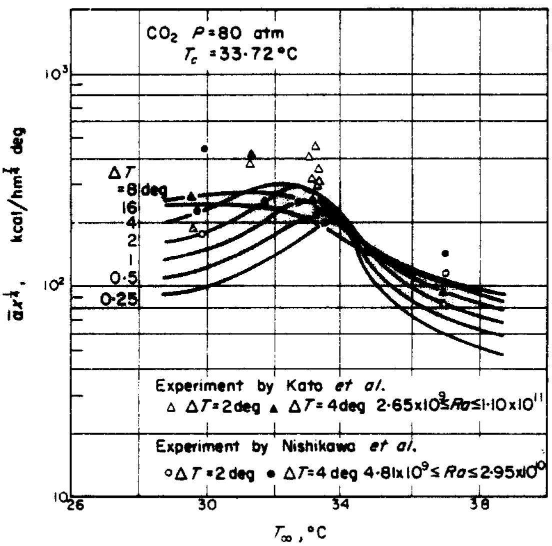

FIG. 9. Relation between $\bar{\alpha}$ and T_∞ (carbon dioxide, 80 atm).

heating wall $x^{\frac{1}{4}}$, as mentioned in Chapt. 2, does not depend on x. So the average Nusselt number was replaced by the following Nusselt number $\overline{Nu}^*$ having a dimension of (length)$^{-\frac{1}{4}}$:

$$\overline{Nu}^* \equiv \overline{Nu}\, x^{-\frac{1}{4}}, \qquad \overline{Nu} \equiv \bar{\alpha}x/\lambda_m \tag{20}$$

Similarly the Rayleigh number was replaced by the following Rayleigh number Ra^* having a dimension of (length)$^{-3}$:

$$Ra^* \equiv Rax^{-3},$$
$$Ra \equiv g(\rho_\infty - \rho_w)\, \rho_m c_{pm} x^3/\lambda_m \mu_m \tag{21}$$

A plot of $\overline{Nu}^*$ vs. Ra^* was compared with the conventional correlation of laminar free-convective heat transfer, which will be transformed as follows:

$$\overline{Nu}^* = 0{\cdot}56\, Ra^{*\frac{1}{4}}. \tag{22}$$

Figure 10 shows the result of water at 230 atm. In the case of a larger temperature difference, therefore, a larger Rayleigh number deviation from the correlation-line becomes larger. Furthermore, it is remarkable that any increase of the temperature difference in the range of large Rayleigh number sometimes results in the decrease of the Rayleigh number and the Nusselt number.

3.3 *Influence of temperature-dependency of each physical property on heat transfer*

Former discussion on convective heat transfer to supercritical fluids seems to have been focused on the anomalous increase of heat-transfer coefficient in some region around the pseudo-critical point. It seems that the sharp peak of specific heat and the very large value of the coefficient of cubic expansion were mostly ascribed to the reason of the singularity. Such an inclination has therefore prevailed that the temperature-dependence of specific heat and density could completely explain the heat transfer to supercritical fluids. For instance, Fritsch and Grosh [1] held the same view, letting the viscosity and the thermal conductivity be equal to those at the mean film temperature in their analysis.

It is, however, needless to say that the viscosity and the thermal conductivity at each local temperature have significant influences on convective heat transfer across a boundary layer. Hence an analysis in which the variable viscosity and thermal conductivity is considered is the step to be followed. A calculation experiment was performed to examine whether there existed such a physical property whose temperature

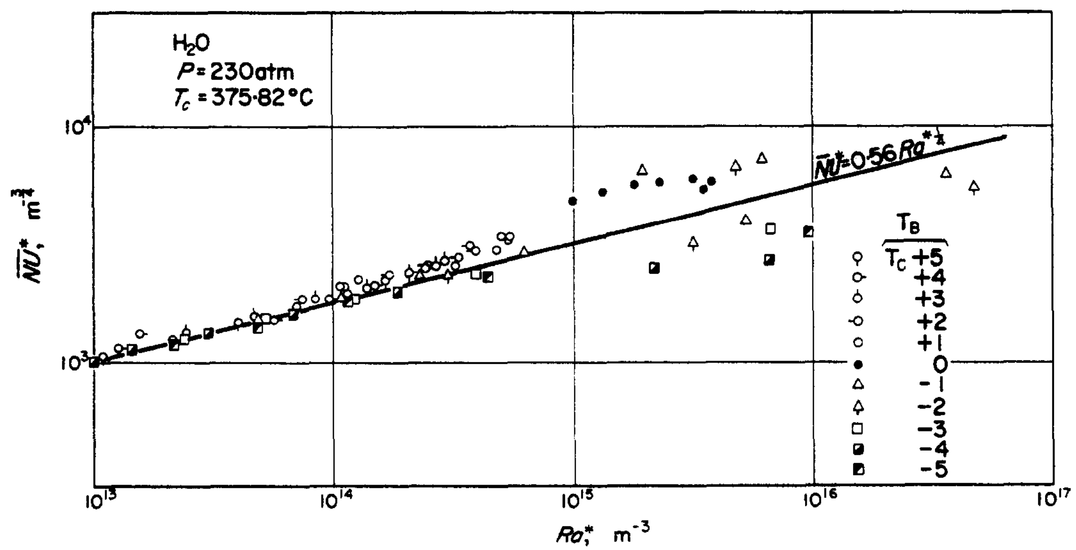

FIG. 10. Nondimensional correlation of the calculated values (water, 230 atm, physical properties at the mean film temperature).

dependency governs heat transfer. Of the four relevant physical properties:

density, specific heat, viscosity and thermal conductivity

density and another one, say specific heat, were treated as variables, while the remainder, say viscosity and thermal conductivity, were fixed at the values corresponding to the mean film temperature. By taking up three kinds of imaginary fluids, calculations were made on water at 240 atm, which were compared with the above-mentioned solution with complete variable physical properties and the Ostrach type solution to a fluid with properties at mean film temperature.

Some examples were shown in Figs. 11–13. In these figures it is remarkable that "CLM" curves are quite similar to "C" curves. The present solution is therefore expected to coincide roughly with that by Fritsch and Grosh [1]. This prediction will be confirmed by several comparisons in the next section. Furthermore, it is worthwhile to note that "L" curves always resemble "M" curves.

3.4. *Comparison with former investigations*

We close our discussion by comparing the present solutions with the former theoretical and experimental studies. Comparisons were made with the theoretical and experimental investigation by Fritsch and Grosh [1, 6] for water, theoretical one by Hasegawa and Yoshioka [2] for water and carbon dioxide,

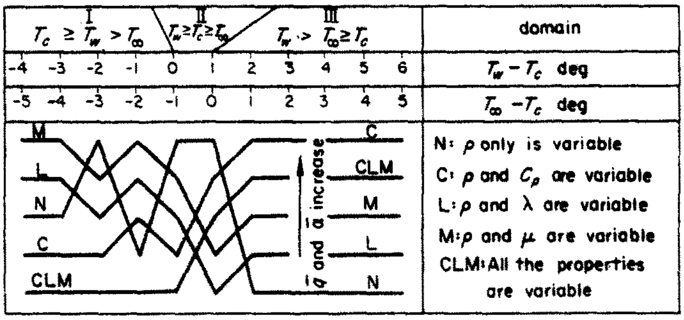

FIG. 11. Influence of temperature-dependency of each physical properties (water, 240 atm, $\Delta T = 1$ deg).

In these figures "CLM" means the above-mentioned solution, "N" the Ostrach type solution and "C", "L" and "M" represent the solutions taking into account of temperature-dependence of specific heat, thermal conductivity and viscosity, respectively. Since in Fig. 11 for fixed temperature differences the maximum deviation among five cases did not exceed 10% in $\bar{q}x^{\frac{1}{4}}$, a heat flux curve was not given. By the way, the maximum deviations in Figs. 12 and 13 were 28 per cent and 54 per cent respectively. Diagrams drawn by polygonal lines in these figures compare magnitudes among the five solutions in $qx^{\frac{1}{4}}$ or $\bar{\alpha}x^{\frac{1}{4}}$, and the upper one shows a larger value than the lower one.

experimental one by Kato *et al.* [7] for carbon dioxide and experimental one by Nishikawa *et al.* [8] for carbon dioxide. Results of comparisons were shown from Figs. 14–18 for water and in Figs. 9, 19 and 20 for carbon dioxide.

At first analytical studies are compared. As mentioned in the previous section theoretical calculation by Fritsch and Grosh which includes only variable density and specific heat shows a favourable accord with the present one. It can be said that for temperature differences less than 4 degC both treatments give practically the same results. A perturbed solution to first order by Hasegawa and Yoshioka shows a remarkable

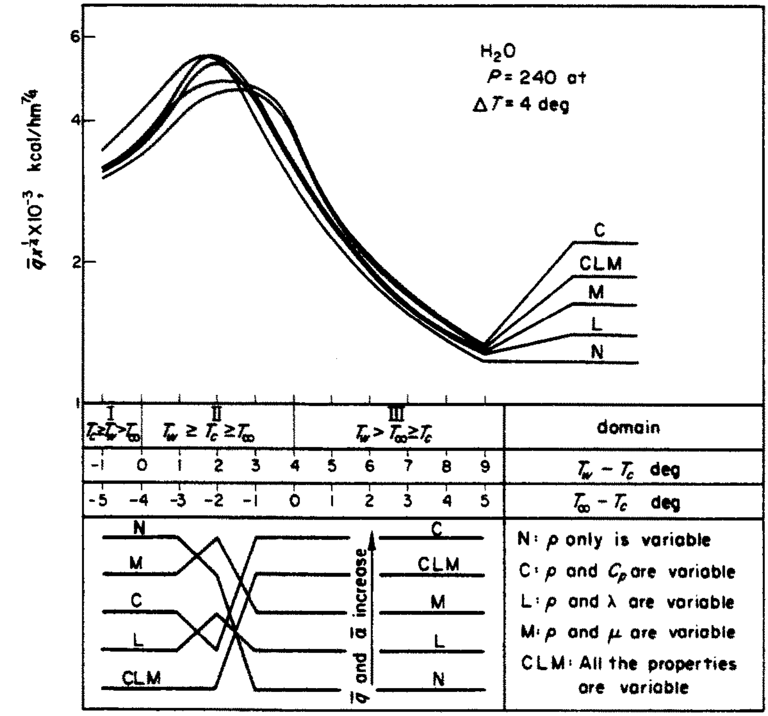

FIG. 12. Influence of temperature-dependency of each physical properties (water, 240 atm, $\Delta T =$ 4 deg).

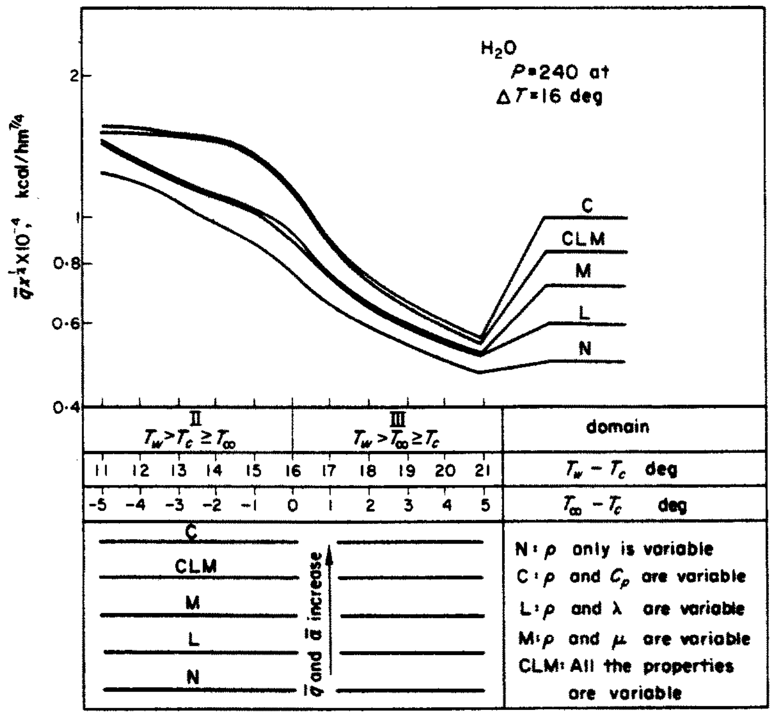

FIG. 13. Influence of temperature-dependency of each physical properties (water, 240 atm, $\Delta T = 16$ deg).

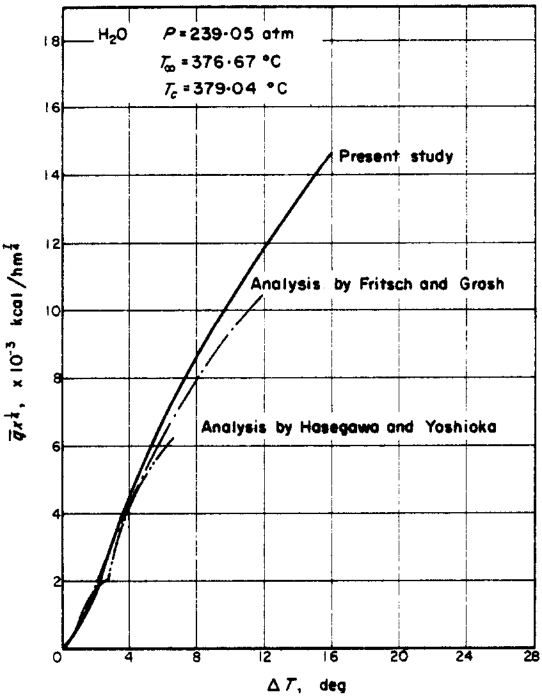

FIG. 14. Comparison with the former investigations (water, 239·05 atm, T_∞ = 376·67°C).

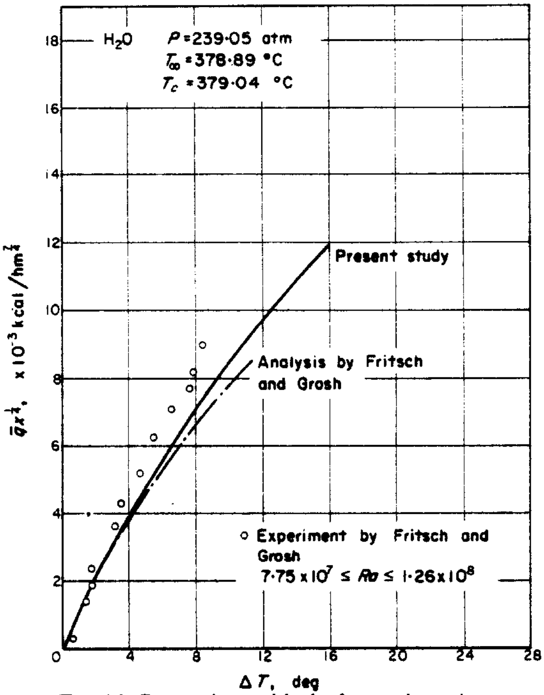

FIG. 16. Comparison with the former investigations (water, 239·05 atm, T_∞ = 378·89°C).

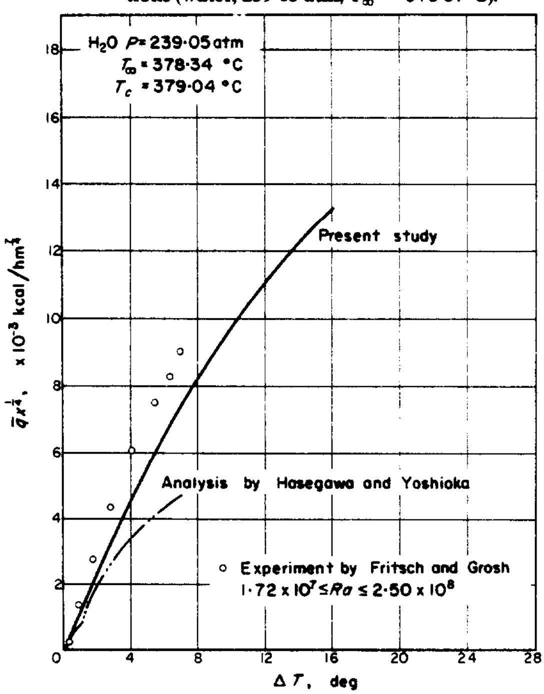

FIG. 15. Comparison with the former investigations (water, 239·05 atm, T_∞ = 378·34°C).

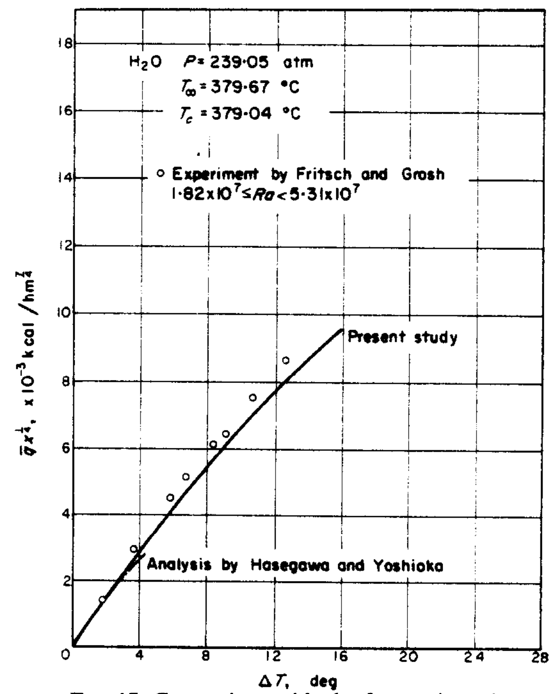

FIG. 17. Comparison with the former investigations (water, 239·05 atm, T_∞ = 379·67°C).

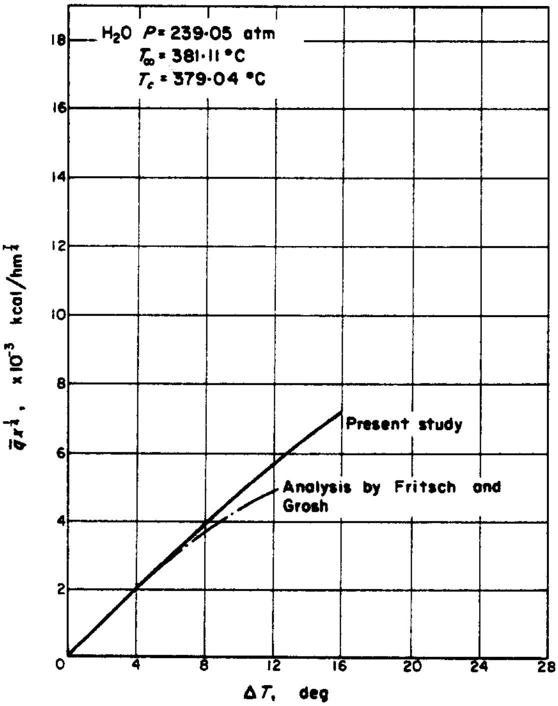

FIG. 18. Comparison with the former investigations (water, 239·05 atm, T_∞ = 381·11°C). Hasegawa and Yoshioka's solution for 240 atm does not differ the present one on the figure for less than 2·89 deg.

coincidence with the present one for small temperature differences.

Secondly the present solution is compared with several experimental investigations. In general, it may be said, they have the tendency to agree with the analysis. There are however some points that make comparison unfeasible. In case of Fritsch and Grosh the chosen height of vertical heating wall ($\frac{1}{8}$ in.) was so small that Rayleigh numbers in their experiment were sufficiently small, but their results could not be said as an experiment for so-called vertical wall. Moreover in cases of Kato *et al.* Rayleigh numbers in their experiments were too large ($10^9 \sim 10^{12}$) to regard their results as the experiments in the laminar flow regime, though the height of vertical walls in their apparatus were fairly large (20 mm). Among other many items which must be examined the technique to measure the temperature of the heating wall were

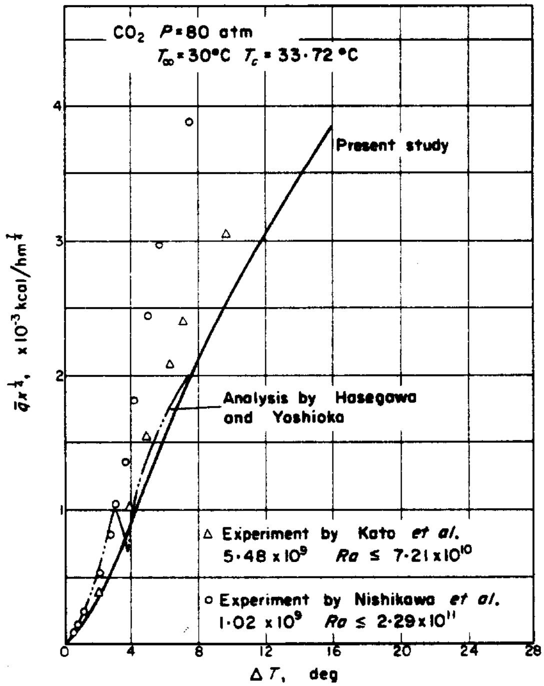

FIG. 19. Comparison with the former investigations (carbon dioxide, 80 atm, T_∞ = 30°C).

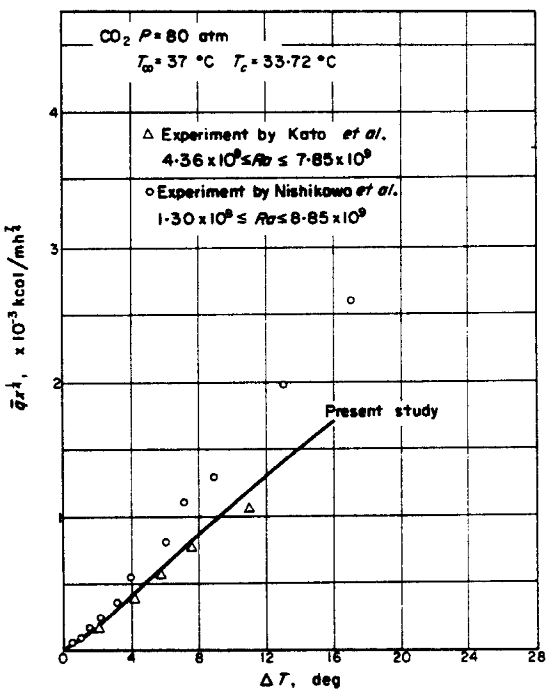

FIG. 20. Comparison with the former investigations (carbon diozide, 80 atm, T_∞ = 37°C).

not suitable in some experiments, and pressure vessel in some apparatus were too small to realize the condition of stagnant fluid in infinitely large space. Accordingly, it would be premature to discuss the validity of analysis by comparison with the available experimental data.

4. SUMMARY

Theoretical studies have been made on the laminar free-convective heat transfer from an isothermal plate to fluids at supercritical pressures by taking into account of the temperature-dependence of all relevant physical properties. The heat transfer characteristics of water at 230, 240 and 250 atm and carbon dioxide at 80 and 90 atm were clarified by integrating the similarity transformed differential equations numerically. Results were as follows:

(1) The heat transfer characteristics are strongly dependent on the bulk fluid temperature and the wall temperature separately.

(2) For the larger temperature difference the conventional equation of laminar free-convective heat transfer fails to correlate the present analytical results.

(3) The analysis by Fritsch and Grosh for temperature-dependent density and specific heat agrees favorably with the present one.

Property-dependence of the heat-transfer characteristics of the supercritical fluids could be said to be thoroughly examined. On the basis of the present study, some idea on the mechanism of heat transfer to supercritical fluids may be obtained.

ACKNOWLEDGEMENT

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ANALYSE DU TRANSPORT DE CHALEUR PAR CONVECTION NATURELLE DANS DES FLUIDES SUPERCRITIQUES À PARTIR D'UNE PLAQUE VERTICALE ISOTHERME

Résumé—Des études théoriques ont été effectuées sur le transport de chaleur par convection naturelle laminaire à partir d'une plaque isotherme vers des fluides à des pressions supercritiques, en tenant compte ont été éclaircies en intégrant numériquement les équations différentielles transformées par similtude. Les caractéristiques de transport de chaleur de l'eau à 230, 240 et 250 atm et du gaz carbonique à 80 et 90 atm ont été éclaircies en intégrant numériquement leséquations différentielles transformées par similitude. Les résultats étaient comme suit:

(1) Les caractéristiques de transport de chaleur sont fortement dépendantes de la température globale du fluide et de la température de la paroi individuellement.

(2) Pour la plus grande différence de température. l'équation classique du transport de chaleur par convection naturelle laminaire n'arrive pas à corréler les résultats analytiques actuels.

(3) L'analyse par Fritsh et Grosh pour la masse volumique et la chaleur spécifique dépendant de la température se compare favorablement avec l'analyse actuelle.

On peut dire que la dépendance des caractéristiques de transport de chaleur des fluides supercritiques en fonctio de leurs propriétés a été examinée entièrement.

Zusammenfassung—Es wurden theoretische Untersuchungen über den laminaren Wärmeübergang bei freier Konvektion von einer isotherm beheizten Platte an Flüssigkeiten bei überkritischen Drücken unter Berücksichtigung der Temperaturabhängigkeit aller verwendeten physikalischen Stoffwerte durchgeführt. Die Wärmeübergangscharakteristiken von Wasser bei 230, 240 und 250 atm und Kohlendioxyd bei 80 und 90 atm wurden durch Integration der ähnlich-transformierten Differentialgleichungen gewonnen. Die Ergebnise waren wie folgt:

(1) Die Wärmeübergangscharakteristiken hängen stark von der Flüssigkeitstemperatur und im einzelnen von der Wandtemperatur ab.
(2) Für eine grössere Temperaturdifferenz versagt die gewöhnliche Gleichung für den Wärmeübergang bei freier Konvektion, um die gefundenen analytischen Ergebnisse zu beschreiben.
(3) Die Untersuchung von Fritsch und Grosh mit temperaturabhängiger Dichte und spezifischer Wärme läasst sich gut mit der vorliegenden Analyse vergleichen.

Es kann gesagt werden, dass die Abhängigkeit der Wärmeübergangskurven für überkritische Flüssigkeiten von den Stoffwerten gründlich untersucht worden ist.

Аннотация—Приводится теоретическое исследование теплообмена при ламинарной естественной конвекции между изотермической пластиной и жидкостями при сверхкритических давлениях с учетом зависимости всех соответствующих физических свойств от температуры. Теплообменные характеристики воды при 230, 240 и 250 атм и двуокиси углерода при 80 и 90 атм выведены при численном интегрировании безразмерных дифференциальных уравнений.

Результаты следующие:

(1) Характеристики теплообмена сильно зависят от температуры стенки.
(2) При большой температурной разности обычное уравнение теплообмена для ламинарной естественной конвекции не учитывает особенности рассмотренного случая.
(3) Зависимости плотности от температуры и удельной теплоты, полученные Фришем и Грошем, хорошо согласуются с данными этой работы.

Таким образом, можно сделать вывод, что тщательно исследовалась зависимость характеристик теплообмена от свойсть жидкостей при сверхкритических параметрах.